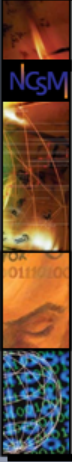

Know Math to teach the Common Core Math-
the story of progressions

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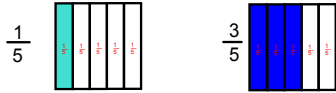
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

National Council of Supervisors of Mathematics
 CCSS Standards of Mathematical Practice: Reasoning and Problem Solving

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3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



(SMP 2, 7)

The importance of specifying the whole

(SMP 6)

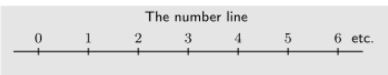
Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.

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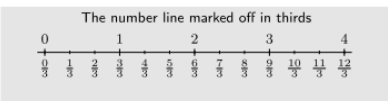
Area representations of $\frac{1}{4}$



In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.



(SMP 4 and SMP 5)



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Using the number line to show that $\frac{4}{3} = \frac{5 \times 4}{5 \times 3}$

$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also 5×4 parts when each part is $\frac{1}{5 \times 3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are 5×3 parts of equal length in the unit interval, and $\frac{4}{3}$ is 5×4 of these. Therefore $\frac{4}{3} = \frac{5 \times 4}{5 \times 3} = \frac{20}{15}$.

(SMP 2,8)

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Representation of $\frac{2}{3} + \frac{3}{5}$ as a length

Using the number line to see that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

Segment of length $\frac{1}{3}$

5 segments put end to end $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

Which SMP's do you see here?

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Using Models to gain understanding of various kinds of fractions

$$\frac{7}{5} + \frac{4}{5} = \frac{1}{5} + \dots + \frac{1}{5} + \frac{1}{5} + \dots + \frac{1}{5}$$

$$= \frac{7+4}{5} = \frac{11}{5}$$

(SMP 2,3)

How would you show? $\frac{1}{5} = \frac{1}{5} = \frac{1}{5} = \frac{1}{5} = \frac{1}{5} = \frac{5}{5} = 1$

It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding fractions.

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4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

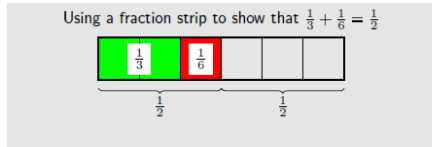
Seeing that $0.2 > 0.09$ using a visual fraction model

The shaded region on the left shows 0.2 of the unit square, since it is two parts when the square is divided into 10 parts of equal area. The shaded region on the right shows 0.09 of the unit square, since it is 9 parts when the unit is divided into 100 parts of equal area.

(SMP 1,3,4)

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5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.



Students discover patterns and then begin to generalize (SMP 7,8)

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5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

For example in the problem

Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes $\frac{1}{2}$ a cup from hers and Lazarus squeezes $\frac{2}{5}$ of a cup from his. How much lemon juice to they have? Is it enough?

students estimate that there is almost but not quite one cup of lemon juice, because $\frac{2}{5} < \frac{1}{2}$. They calculate $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$, and see this as $\frac{1}{10}$ less than 1, which is probably a small enough shortfall that it will not ruin the recipe. They detect an incorrect result such as $\frac{2}{5} + \frac{2}{5} = \frac{3}{7}$ by noticing that $\frac{3}{7} < \frac{1}{2}$.

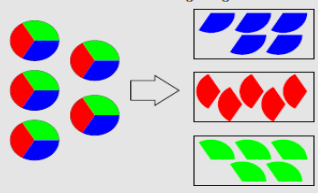


(SMP 1, 2, 3, 4, 5, 6)

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5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

How to share 5 objects equally among 3 shares:
 $5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

(SMP 3)

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